CHAPTER 8: Dynamics II: Motion in a plane — March 14, 19

↳ Really just more Newton's Laws for Circular Motion

Centripetal Acceleration → acceleration because of a change in direction (not speed) for an object moving at speed v in a circular path of radius r

$$a = v^2/r$$

* direction is towards the center of the circular path.

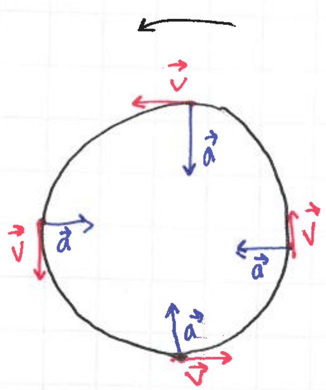
Uniform circular motion → Constant Speed v, Constant radius r

$$v = \frac{2\pi r}{T}$$

r → radius in m

T → period in seconds

↳ $\frac{sec}{rev}$



* $\vec{v}$ is always tangent to the path
* $\vec{a}$ is always toward the center (if speed is constant).

if r = 0.50 m and object is rotating at 75 mph, what is the speed v at the period T?

$$\frac{75\,rev}{min}\left[\frac{2\pi(0.50m)}{1\,rev}\right]\left[\frac{1\,min}{60\,sec}\right] = 3.93\,m/s \rightarrow \boxed{v = 3.93\frac{m}{s}}$$

$$\frac{1\,min}{75\,rev}\left(\frac{60s}{1\,min}\right) = \frac{0.80s}{rev} \rightarrow \boxed{T = 0.80s}$$

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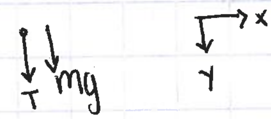
continued...

$$v = \frac{2\pi r}{T} = \frac{2\pi(0.50m)}{0.80s} = 3.93m/s$$

Ex.

A 2.0kg ball is swung in a vertical circle of radius 1.0m at a speed of 5.0m/s. What is the tension in the string at the highest & lowest point. (No air resistance)

Top:



(define down as +)

$\Sigma F_y = ma_y$ $\quad a_y = \frac{v^2}{r}$

$T + mg = ma_y$

$T + mg = \frac{mv^2}{r}$

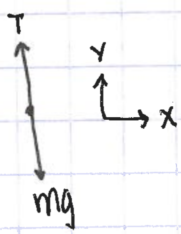
$T = m\left(\frac{v^2}{r}\right) - mg$

$T = m\left(\frac{v^2}{r} - g\right)$

$T = 2.0kg\left(\frac{(5.0m/s)^2}{1m} - 9.80m/s^2\right)$

$T = 30.4N$

lowest point:



$\Sigma F_y = ma_y$ $\quad a_y = \frac{v^2}{r}$

$T - mg = ma_y$

$T = m\left(\frac{v^2}{r}\right) + mg$

$T = m\left(\frac{v^2}{r} + g\right)$

$T = 2.0kg\left(\frac{(5.0m/s)^2}{1m} + 9.80m/s^2\right)$

$T = 69.6N$

- $T = mv^2/r - mg$
  - What is the minimum speed required to maintain a circular path?

* ↳ Tension must be >0

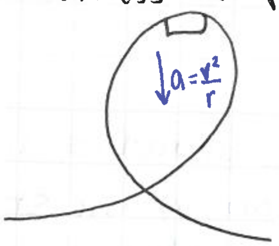
$T = \frac{mv^2}{r} - mg > 0$

$\frac{mv^2}{r} > mg \rightarrow \frac{v^2}{r} > g$

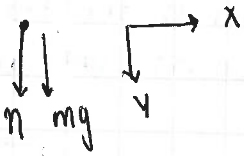
$v^2 > rg \rightarrow v > \sqrt{rg}$

Critical Speed: Vertical: $\sqrt{rg}$ ← speed required to h 8 March 14, 2018

ctionless loop-the-loop



What is the Vmin at top so car stays on the track?



$$\sum F_y = ma_y$$

$$n + mg = m\frac{v^2}{r}$$

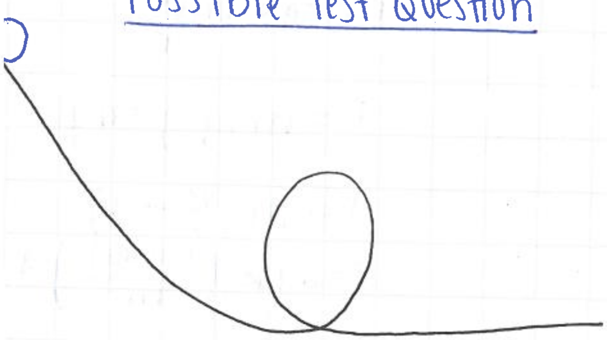
$$n = m\frac{v^2}{r} - mg$$

$$n > 0$$

$$m\frac{v^2}{r} - mg > 0$$

$$v > \sqrt{rg}$$

Possible Test Question



How high must ball be released from to do a complete loop-to-loop?

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*when doing circular motion problems, rather than x-y-z-axis it is useful to define 3 new axes:

r-axis (radial axis) → points toward the center of the circular path.

$$\sum F_r = \frac{mv^2}{r}$$

t-axis (tangential axis) → tangent to the circular path

$\sum F_t = 0$ if speed is constant

$\sum F_t = m\frac{d\vec{v}}{dt}$ Speed is changing

z-axis → Perpendicular to the plane of motion

$\sum F_z = 0$

Radial acceleration $a_r = \frac{v^2}{r}$ towards center

tangential acceleration $a_t = \frac{d\vec{v}}{dt}$ direction is Same as $\vec{v}$, if speeding up
opposite as $\vec{v}$, if Slowing down

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RECAP

4/18/2019

Centripetal (Radial) acceleration → $a = \frac{v^2}{r}$

*direction is towards center of circular path

$V = \frac{2\pi r}{T}$

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* rtz-coordinate System
origin is at location of particle

$r \rightarrow$ towards center
$t \rightarrow$ tangent to path
$z \rightarrow \perp$ to plane of motion

$$\Sigma F_r = ma_r = \frac{mv^2}{r}$$

$$\Sigma F_t = ma_t$$

$$\Sigma F_z = ma_z = 0$$

a_r (change in direction) and a_t (change in speed) are $\perp$ to each other

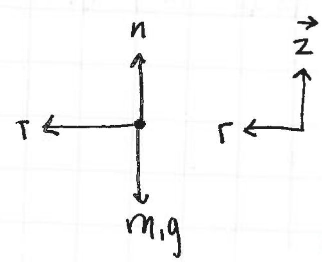
$$|\vec{a}_{total}| = \sqrt{a_t^2 + a_r^2}$$

Critical speed $\rightarrow$ $v = \sqrt{rg}$
↳ slowest speed at which car can complete circle
→ solve for when $n = 0$

Problem 8.13

mass m_2

$\Sigma F_y = ma_y = 0$
$T - m_2g = 0$
Eq 1: $T = m_2g$



$$\Sigma F_r = ma_r = \frac{mv^2}{r}$$

Eq 2: $T = \frac{m_1v^2}{r}$

put (1) $\rightarrow$ (2)

$m_2 g = m_1 v^2/r$

$$v^2 = \frac{m_2rg}{m_1}$$

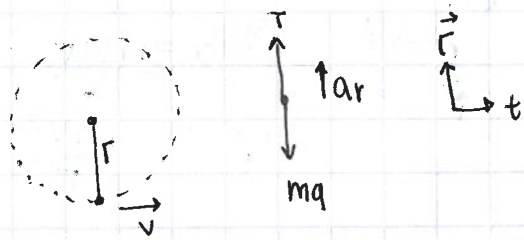
$$v = \sqrt{\frac{m_2rg}{m_1}}.$$

A rock is whirled on the end of a string in a horizontal circle of radius R with a constant period. If the radius of the circle is reduced to R/2, while the period remains T, what happens to the centripetal acceleration of the rock?

$$a = \frac{4\pi^2 r_1}{T_1^2} \qquad a_2 = \frac{4\pi^2 r_2}{T_2^2} = \frac{4\pi^2 \left(\frac{r_1}{2}\right)}{T_1^2} = \frac{1}{2}\left[\frac{4\pi^2 r_1}{T_1^2}\right]$$

$$= \frac{1}{2} a_1$$

Problem 8.24



$$\Sigma F_r = ma_r = \frac{mv^2}{r}$$

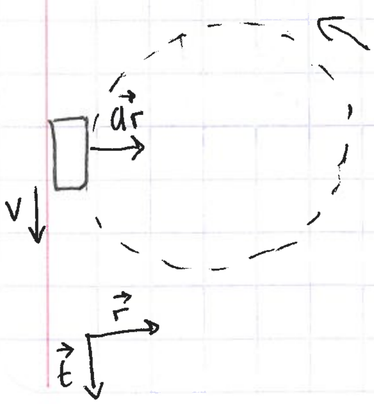
$$T - mg = \frac{mv^2}{r}$$

$$v^2 = \frac{r(T-mg)}{m}$$

$$v = \sqrt{\frac{(1.5\,m)[15\,N - (0.50\,kg)(9.80\,m/s^2)]}{0.50\,kg}} \qquad \boxed{v = \sqrt{\frac{r(T-mg)}{m}}}$$

$$\boxed{v = 5.50\,m/s^2}$$

Problem 8.29



$\Sigma F_r = ma_r = \frac{mv^2}{r}$ to the right

$\Sigma F_t = ma_t \qquad a_t = 12\,m/s^2$ straight downward

$$\Sigma F_r = \frac{mv^2}{r} = \frac{(85{,}000\,kg)(55\,m/s)^2}{130\,m}$$

$$= 1.978 \times 10^6\,N \longrightarrow$$

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$\sum F_t = ma_t = (85000 kg)(12 m/s^2) = 1.02 \times 10^6 N \downarrow$

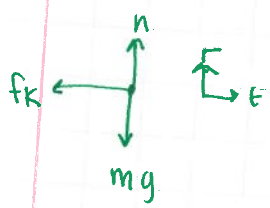
$|\vec{F}_{net}| = \sqrt{(1.978 \times 10^6 N)^2 + (1.02 \times 10^6 N)}$

$= \boxed{2.2 \times 10^6 N}$

$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$

$\theta = \tan^{-1}\left(\frac{-1.02 \times 10^6 N}{1.978 \times 10^6 N}\right) = \boxed{-27^\circ}$

FOR HW- problem 30



B)

$\sum F_r = ma_r$

$n - mg = \frac{mv^2}{r}$

c)

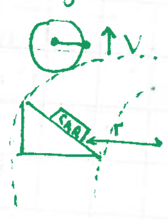
$\sum F_r = ma_r$

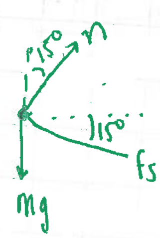
$mg - n = \frac{mv^2}{r}$

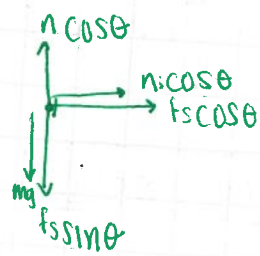
Problem 8.56 figure out speed & proj. motion

Problem 8.59

Problem 8.40







$\sum F_z = ma_z = 0$

$n\cos\theta - mg - f_s \sin\theta = 0$

$\sum F_r = ma_r = \frac{mv^2}{r}$

$n\sin\theta + f_s \cos\theta = \frac{mv^2}{r}$

$f_s = f_{s,max} = \mu_s n$